Topic: Trigonometric limits

Question: Find the limit.

$$\lim_{x \to 0} \cos \frac{1}{x}$$

Answer choices:

A 0

B -1

C 1

D Does not exist (DNE)

© Krista King Math

Solution: D

This function continuously oscillates between $+1$ and -1 as x approaches 0. Therefore, it's impossible to pinpoint a limit at $x = 0$, and therefore the limit does not exist.

© Krista King Math

Topic: Trigonometric limits

Question: Find the limit.

$$\lim_{x \to 0} \frac{7}{x \csc x}$$

Answer choices:

A 0

B 7

C -7

D ∞

© Krista King Math

Solution: B

We can't evaluate this limit by direct substitution because we would get zero in the denominator. We need to rewrite the function so that we can find the limit. We know that

$$\csc x = \frac{1}{\sin x}$$

and so we can rewrite the function as

$$f(x) = \frac{7}{x \csc x}$$

$$f(x) = \frac{7}{x\left(\frac{1}{\sin x}\right)}$$

$$f(x) = \frac{7}{\frac{x}{\sin x}}$$

$$f(x) = 7 \cdot \frac{\sin x}{x}$$

$$f(x) = \frac{7 \sin x}{x}$$

Recall the special trigonometric limit

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Therefore,

© Krista King Math

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{7\sin x}{x}$$

$$7\lim_{x\to 0} \frac{\sin x}{x}$$

$$7(1)$$

$$7$$

© Krista King Math

Topic: Trigonometric limits

Question: Find the limit.

$$\lim_{h \to 0} \frac{\cos h - 1}{h}$$

Answer choices:

A 0

B 1

C -1

D ∞

© Krista King Math

Solution: A

Recall the special trigonometric limit

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

This is very close to our original problem, and all we have to do is factor -1 out of the numerator in order to have the special limit.

$$\lim_{h \to 0} \frac{\cos h - 1}{h}$$

$$\lim_{h \to 0} - \frac{1 - \cos h}{h}$$

$$-0$$

$$0$$

© Krista King Math